lows that

$$\frac{(1-\bar{b})\left(1+\exp\left[-\frac{\pi\bar{L}\alpha}{\bar{a}+\bar{b}}\right]\right)}{1+\frac{\alpha^2\bar{L}^2}{(\bar{a}+\bar{b})^2}} = \frac{(1-\bar{a})\left(1+\exp\left[-\frac{\pi\bar{L}(\alpha+1)}{\bar{a}+\bar{b}}\right]\right)}{1+\frac{\bar{L}^2(\alpha+1)^2}{(\bar{a}+\bar{b})^2}} \tag{4}$$

which is an equation relating the dimensions of the channel. Equation (4) is equivalent to the "rule of thumb" in Ref. 1 and for the mean channel height large compared to the amplitude of the sidewall contraction, Eq. (4) reduces to the rule of thumb in Ref. 1.

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Characteristics of a Magnetic Annular Arc Operating Continuously at Atmospheric Pressure

G. W. Garrison* and R. T. Smith*
ARO Inc., Arnold Air Force Station, Tenn.

Magnetic Annular Arc (MAARC) devices have been operated continuously in the millibar-kilowatt range[1] and have been pulsed in the atmosphere-multimegawatt range.[2,3] The AEDC MAARC, a modified version of a device designed for low-pressure operation,[4] was operated on nitrogen exhausting to the atmosphere to determine performance trends. The electrical and thermal characteristics of this direct-current, one-half megawatt MAARC are discussed.

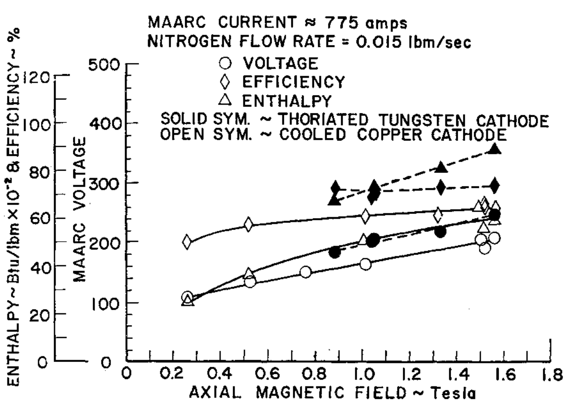

Fig. 1 Experimental data showing the effect of thoriated tungsten cathode and copper cathode on performance as the axial magnetic field is increased.

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* Research Engineer, Propulsion Wind Tunnel Facility. Member AIAA.

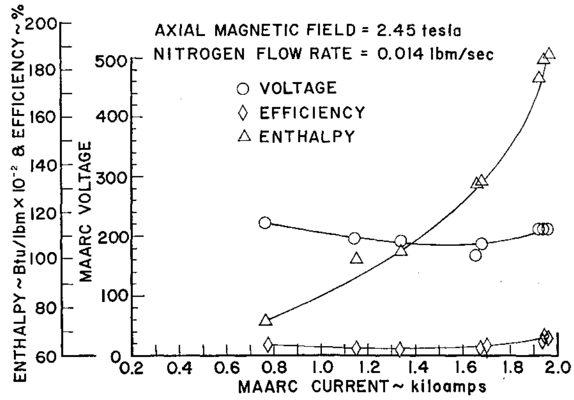

Fig. 2 The variation with current of the voltage, enthalpy, and thermal efficiency at a high axial magnetic field.

The MAARC consists of a constant area coaxial copper anode, 3.4 cm in diameter, and cathode, 2.5 cm in diameter. The cathode tip was hemispherical and both copper and thoriated tungsten were used. A primarily axial magnetic field up to 2.5 tesla (25,000 gauss) was applied at the cathode tip by an external magnet. The ratio of the radial magnetic field to the axial magnetic field in the discharge was approximately 0.11. A high-speed movie camera was used to photograph the discharge at 10,000 frames/sec with a shutter speed of two μsec. All components of the MAARC and magnet were water cooled allowing continuous operation at arc currents from 700 to 2000 amp.

Figure 1 shows the variation of enthalpy, voltage, and efficiency with increasing magnetic field for both the copper and tungsten cathodes. Operation with a tungsten cathode invariably gives a higher voltage and efficiency, thus a higher enthalpy. The cathode heat loss using tungsten is approximately one-half of that for copper. This may be attributed to the thermal emission of the tungsten tending to cool the cathode. The over-all efficiency of the device increases with magnetic field due to the decrease of the fraction of the power lost to the cathode and anode. The voltage, for both materials, increases linearly with increasing magnetic field agreeing with earlier experimental results.[1–3]

The MAARC has a drooping V-I characteristic except at high current and high magnetic fields where the slope becomes positive. This voltage characteristic, as well as the variation of enthalpy and efficiency with current, is shown in Fig. 2. The anode power loss increases linearly with increasing current, but the cathode loss is independent of current. The over-all efficiency of the device decreases slightly and then increases as the current increases. The enthalpy changes rapidly with increasing current and the maximum enthalpy measured (based on a heat balance) during the

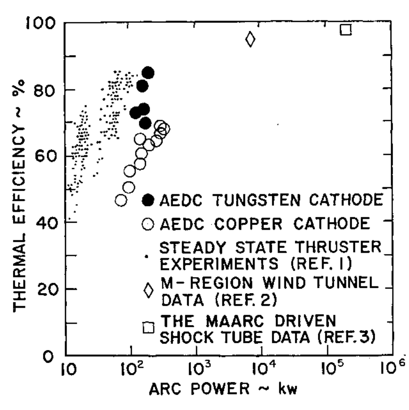

Fig. 3 Comparison of the thermal efficiency of the AEDC steady-state atmospheric MAARC with the data from the low-pressure and pulsed experiments.

series of tests was 18,000 Btu/lbm. This is not a MAARC limit but a system limit.

As the mass flow increases, the voltage and fraction of power lost to the cathode is constant or increases slightly. The anode loss decreases with increasing mass flow, thus the over-all efficiency of the device increases.

Figure 3 shows the variation of the over-all efficiency of the device with total power and includes data from other facilities. These earlier data indicate that the efficiency increases with power achieving an efficiency above 95% at the 100 Mw level. This trend is verified in the present device using both copper and tungsten cathodes. An efficiency of 70% is obtained at 0.4 Mw using the copper cathode, and 85% is obtained at 0.2 Mw using the tungsten cathode.

Three sets of consecutive frames taken from the high-speed films of the discharge are shown in Fig. 4. Each set is typical of the discharge at the operating conditions specified. Set "a" was taken at a field strength of $B = 0.265$ tesla, current $I = 760$ amp, and a mass flow of $\dot{m} = 0.014$ lbm/sec. At this condition, the discharge consists of a concentrated current path, or "spoke," and is typical of the discharge in conventional arc heaters. The spoke rotated at a measured frequency of 3180 Hz. Assuming that the spoke rotates with the bulk gas, an expression for the rotational frequency can be derived as $\omega = IB/\dot{m}2\pi(r_o^2 - r_i^2)/(r_o^2 + r_i^2)$ where r_i and r_o are the inner and outer electrode radii, respectively. This yields a frequency of 1509 Hz for this device. If the discharge occurs at the cathode tip, then $r_i \sim 0$ and one obtains a frequency of 5060 Hz. These values bracket the measured frequency indicating that the discharge originates somewhere between the tip and the outer surface of the cathode.

In order for the discharge to completely fill the annulus between cathode and anode, i.e., become diffuse, the time for the spoke to make one rotation ($\tau_{\text{rot}} = 1/\omega$) must be much less than the residence time (τ_{res}) and/or the characteristic time for recombination (τ_{rec}). For the conditions of this experiment $\tau_{\text{rec}} \approx 1{,}000$ μsec, $\tau_{\text{res}} \approx 200$ μsec, and for condition "a" $\tau_{\text{rot}} \approx 400$ μsec. The criteria for a diffuse discharge are not met.

The visible current path appears to fluctuate between a spoke and a diffuse discharge for the conditions of set b. Although the time to make one revolution is 70 μsec, the current is apparently not high enough to sustain a steady diffuse discharge.

The calculated rotational time for set c is 30 μsec, which is less than the recombination and the residence time, and the necessary conditions for a diffuse discharge are present.

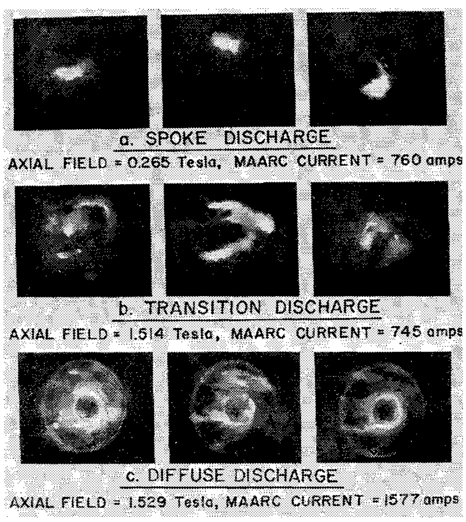

Fig. 4 Consecutive high-speed photographs of the MAARC discharge between the center cathode and coaxial anode (frame rate ≈10,000 frames/sec, shutter speed ≈2 μsec, copper electrodes).

Also, the camera will stop a spoke rotating at 500,000 Hz ($\tau_{\text{rot}} \approx 2$ μsec) but no confirmed current path is detected. Hence, the discharge shown in set c is considered diffuse.

Although the magnetic fields utilized were relatively low, the performance obtained with the MAARC indicated that successful operation at atmospheric pressure is possible. The basic criteria are fulfilled for the operation of a MAARC providing supersonic flow with high stagnation pressure and high enthalpy. A MAARC of improved design is being fabricated with the goal of operation at these conditions.

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Bending Stress at a Clamped Support of an Impulsively Loaded Conical Shell

D. B. LONGCOPE* AND M. J. FORRESTAL†
Sandia Laboratories, Albuquerque, N. Mex.

IN this Note, a perturbation solution is developed for the bending stress at a clamped support of a semi-infinite, conical shell subjected to a uniform, normal impulse. The response is described by the shell-bending theory which neglects transverse-shear deformation and rotary inertia. In Ref. 1, this theory was shown to adequately predict the maximum bending stress at a clamped support of an impulsively loaded, circular, cylindrical shell.

The small perturbation parameter is $\sin\alpha$ where α is the half apical angle shown in Fig. 1. It is shown that the bending stress history for a conical shell with a small half apical angle closely follows the bending stress history for a circular, cylindrical shell.

Shell Equations of Motion

Equations for the axisymmetric motion of a conical shell are recorded in Ref. 2. With the slant coordinate x defined

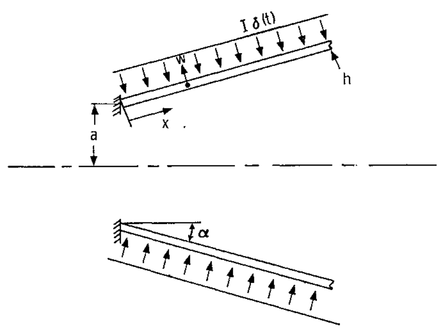

Fig. 1 Geometry of the problem.

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* Staff Member.

† Staff Member. Member AIAA.